Comment on Pasti – Sorokin – Tonin approach to three – brane [1]

Alexei Nurmagambetov [2]

Kharkov Institute of Physics and Technology
310108, Kharkov, Ukraine

Abstract

We construct a manifestly dual formulation of Dirichlet three – brane in the framework of Pasti – Sorokin – Tonin approach.

PACS: 11.15 - q, 11.17 + y

Keywords: p – branes, duality, Born – Infeld theory.

[1]Work supported in part by the INTAS Grant No 93-493-ext, and by Ukrainian State Committee on Science and Technology under the contract No 2.3/664.

[2]e-mail: alesha@susy-1.kipt.kharkov.ua

Recently, the number of papers concern to duality in Born – Infeld theory have appeared in literature (see for example [1], [2] and Refs. therein). Following to the previously obtained results of Refs. [3], [4] noted the self – duality of Dirichlet (super) – three – brane, the authors of [1], [2] consider the Born – Infeld electrodynamics, being the worldvolume field theory of D(irichlet) three – brane, in a spirit of Schwarz – Sen construction [5], covariantized in the framework of Pasti – Sorokin – Tonin (PST) approach [6]. In this note I make a sketch of PST formulation for bosonic D(irichlet) – three – brane and give some comments concerning its supersymmetric extension.

Our starting point is the following action for D(irichlet) three – brane:

$$S = \int d^4\xi \sqrt{-\det(g_{mn} + F^{\alpha}_{mn})} - \sqrt{-g}\frac{1}{4\partial_l a \partial^l a}\partial^m a \mathcal{F}^{\alpha}_{mn}\mathcal{F}^{\alpha\ np}\partial_p a, \tag{1}$$

where, following to the [5] and [6], we introduce two abelian fields $A^{\alpha}_m(\xi)$, $(\alpha = 1, 2)$ to make the electric – magnetic duality manifest at the level of action. Then, F^{α}_{mn} is the field strength $F^{\alpha}_{mn} = 2\partial_{[m}A^{\alpha}_{n]}$ and

$$\mathcal{F}^{\alpha}_{mn} = \mathcal{L}^{\alpha\beta}F^{\beta}_{mn} - F^{*\alpha}_{mn} = \frac{1}{2}\epsilon_{mnpq}\mathcal{L}^{\alpha\beta}\mathcal{F}^{\beta\ pq} \tag{2}$$

with $\mathcal{L}^{\alpha\beta} = -\mathcal{L}^{\beta\alpha}$, $(\mathcal{L}^{12} = 1)$, $F^{*\alpha}_{mn} = \frac{1}{2}\epsilon_{mnlp}F^{\alpha\ lp}$ (see [5], [6] for details). The presence of the uncontracted index α in the first term of (1) seems incorrect, but this is rather the author notation, because

$$\sqrt{-\det(g_{mn} + F^{\alpha}_{mn})} = \sqrt{-g}\sqrt{1 - \frac{1}{2}F^{\alpha}_{mn}F^{\alpha\ mn} - \frac{1}{16}(F^{*\ \alpha}_{mn}F^{\alpha\ mn})^2}.$$

The action (1), written in manifestly covariant form, is invariant under:

- worldvolume diffeomorphisms,
- usual gauge invariance

$$\delta A^{\alpha}_m = \partial_m \phi^{\alpha}, \tag{3}$$

- transformations of the form

$$\delta A^{\alpha}_m = \partial_m a(\xi)\phi^{\alpha}(\xi), \tag{4}$$

- and additional local symmetry

$$\delta a(\xi) = \Phi(\xi), \quad \delta A^{\alpha}_m = \frac{2\Phi(\xi)}{\partial_l a \partial^l a}\mathcal{L}^{\alpha\beta}\mathcal{F}^{\beta}_{mn}\partial^n a, \tag{5}$$

being crucial for establishing a connection to the non - covariant formalism of Refs. [1], [2]. It can be achieved by fixing a gauge, say

$$\partial_m a(\xi) = \delta^3_m. \tag{6}$$

Because of completely auxiliary role of $a(\xi)$ variable, whose equation of motion does not lead to a new field equation in general [6] and is the identity for the case, we can eliminate it from the action without losing dynamical information.

The symmetry (4) allows one to reduce the general solution for the equations of motion of A^{α}_{m} fields to the form

$$\mathcal{V}^{\alpha}_{mn} - \frac{1}{2}\mathcal{F}^{\alpha}_{mn} = 0, \tag{7}$$

$$\mathcal{V}^{\alpha\ mn} = \frac{\delta\sqrt{-\det(g_{mn} + F^{\alpha}_{mn})}}{\delta F^{\alpha}_{mn}},$$

being a generalization [7] of the self – duality condition

$$\mathcal{F}^{\alpha}_{mn} = 0. \tag{8}$$

It is straightforward to find a modification of the action (1) in a background of antisymmetric gauge fields of D=10 supergravity, or, equivalently, in the presence of external sources. Following to the enlightening paper of Medina and Berkovits [8], we have to replace the field strength F^{α}_{mn} with

$$H^{\alpha}_{mn} = F^{\alpha}_{mn} - C^{\alpha}_{mn}, \tag{9}$$

$$\mathcal{H}^{\alpha}_{mn} = \mathcal{L}^{\alpha\beta} H^{\beta}_{mn} - H^{*\ \alpha}_{mn} \tag{10}$$

and add to the action (1) a Wess – Zumino term (see, for instance, [3], [4], [9]). The resulting action becomes

$$S = \int d^4\xi\, \sqrt{-g}\Big(\sqrt{1 - \frac{1}{2}H^{\alpha}_{mn}H^{\alpha\ mn} - \frac{1}{16}(H^{*\ \alpha}_{mn}H^{\alpha\ mn})^2} - \frac{1}{4\partial_l a \partial^l a}\partial^m a \mathcal{H}^{\alpha}_{mn}\mathcal{H}^{\alpha\ np}\partial_p a\Big)$$

$$+ \int_{\mathcal{M}^4} (C^{(4)} + \frac{1}{2}\mathcal{L}^{\alpha\beta} F^{\alpha\ (2)} \wedge C^{\beta\ (2)} + \theta F^{\alpha\ (2)} \wedge F^{\alpha\ (2)}), \tag{11}$$

where $C^{(4)}$ and $C^{(2)}$ are pullbacks of the corresponding D=10 forms onto worldvolume $\mathcal{M}^4$.

The structure of the Wess – Zumino term is governed by the requirement of invariance of (11) under the modified by $\mathcal{F}^{\alpha}_{mn} \to \mathcal{H}^{\alpha}_{mn}$ symmetries (4) and (5), that is, the Wess – Zumino term is required to preserve local symmetries of the action when three – brane couples to the antisymmetric fields (see [9], where this fact was pointed out for the first time).

Thus, we have constructed a manifestly dual formulation of three – brane coupled to the antisymmetric gauge fields of D=10 supergravity in the framework of PST approach.

In conclusion I would like to note that from the point of view of supersymmetric extension of this approach to the case of IIB D=10 self - dual D(irichlet) three – brane, it turns out to be possible at least from the matching bosonic and fermionic degrees of freedom of the model. Indeed, we have six bosonic degrees of freedom, coming from

the transverse excitations of three – brane and eight fermionic degrees of freedom on the mass - shell. The rest is the two bosonic degrees of freedom, coming from the worldvolume self - dual vector fields A_m^α. It would be interesting to extend the results obtained here to the supersymmetric case.

Acknowledgements. The author is grateful to Dima Sorokin for illuminating discussions of the details of PST approach and to Igor Bandos for useful remarks.

References

[1] D.Berman, Preprint **hep-th/9706208**.

[2] A.Khoudeir and Y.Parra, Preprint **hep-th/9708011**.

[3] A.Tseytlin, *Nucl. Phys.* **B469** (1996) 51.

[4] M.Aganagic, J.Park, C.Popescu and J.Schwarz, *Nucl. Phys.* **B496** (1997) 215, Preprint **hep-th/9702133**.

[5] J.Schwarz and A.Sen, *Nucl. Phys.* **B411** (1994) 35.

[6] P.Pasti, D.Sorokin and M.Tonin, *Phys. Lett.* **352B** (1995) 59; *Phys. Rev.* **D52** (1995) 4277.

[7] M.Perry and J.Schwarz, *Nucl. Phys.* **B489** (1997) 47.

[8] R.Medina and N.Berkovits, Preprint **hep-th/9704093**.

[9] P.Pasti, D.Sorokin and M.Tonin, *Phys. Lett.* **398B** (1997) 41, Preprint **hep-th/9701037**.